**CSC236 Assignment1**

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Question 1

To proof this question by simple induction, we need to use one already-known claim.

Claim 1: For any arbitrary graph G = (V, E), I can always choose a vertex with

edges incident with it.

(a)

Answer: Yes.

Proof: Assume P(234)

That is, for any arbitrary bipartite graph G = (V, E) with |V| = 234, G has no more

than edges.

WTS P(235) follows.

Let be an arbitrary bipartite graph with || = 235.

By Claim 1, I can always choose a vertex in with edges incident

with it.

Without losing generality, choose this vertex, call it .

Let be a bipartite graph s.t.

Hence |

By hypothesis P(234), we know that has no more than edges.

Hence

We have proved that P(235) follows. ￭

(b)

Answer: No.

Reason: Assume P(235)

That is, for any arbitrary bipartite graph G = (V, E) with |V| = 235, G has no more

than edges.

If we want to show P(236) follows:

Let be an arbitrary bipartite graph with || = 236.

By Claim 1, I can always choose a vertex in with edges

incident with it.

Without losing generality, choose this vertex, call it .

Let be a bipartite graph s.t.

Hence |

By hypothesis P(235), we know that has no more than edges.

Hence

However, we CANNOT get

Hence we cannot prove P(236) follows by assuming P(235) holds.

(c)

Strengthen P(n), call it R(n): Every bipartite graph on n vertices has no more than edges

Proof of R(n): (Simple Induction)

Base case: n = 0

Graph with o vertex has 0 edges

P(0) holds.

Inductive step: Let

Assume R(n) holds, i.e. for any arbitrary bipartite graph G = (V, E) with

|V| = n, G has no more than edges.

WTS that R(n+1) also holds

Let be an arbitrary bipartite graph with || = n+1.

By Claim 1, I can always choose a vertex in with edges incident with

it.

Without losing generality, choose one of this vertex, call it .

Let be a bipartite graph s.t.

Hence |

By hypothesis P(n), we know that has no more than edges.

Then we need to show:

Let , we have two cases to consider: n = 2k and n = 2k+1, namely n is even

Or n is odd.

Case n = 2k:

Case n = 2k+1:

In both cases we have proved that , Hence R(n+1) follows. ￭

We use simple induction proved R(n), since , we know that R(n) implies P(n).

Hence P(n) is also True for all natural number n.

Question 2

(a)

Answer: Yes.

Proof: Assume P(3), that means f(3) is a multiple of 4 i.e. f(3) = 4k for some .

Let k be this number.

We want to show that P(29) follows.

(since )

(since P(3) and for some , f(3) = 4k)

(namely, )

Hence f(29) is a multiple of 4 and P(29) follows.  ￭

(b)

Answer: No.

Reason: Assume P(4) holds, i.e. f(4) is a multiple of 4

For P(29)

(since )

There is no direct connection between f(29) and f(4), so we cannot prove P(29) follows

by assuming P(4) holds.

(c)

Proof: (Complete induction)

Base Case: ① n = 1

(since f(0) = 3 and )

12 =

P(1) holds.

② n = 2

(since f(0) = 3 and )

12 =

P(2) holds.

Inductive Step: Assume P(i) holds for

We want to show: P(n) holds for .

Also, since

()

(, since and by induction hypothesis)

Hence *f*(n) is a multiple of 4

P(n) follows ￭

Question 3

To prove this question, we need to use an already-known claim.

Claim: if a prime number divides a perfect cube , then also decides n.

Proof: (Contradiction)

Assume (for the sake of contradiction)

Let set ,then ,

and is non-empty (by assumption).

By Well-Ordering Principle, has a smallest element, call it .

By the definition of ,

Notice that

Take back to the original equation, we get

Notice that

Take back to the original equation, we get

Notice that

Take back to the original equation, we get

It is obvious that should be in set S, and , which gives us a contradiction

Conclusion: no such integers satisfy . ￭

Question 4

(a)

Define P(t):

Claim:

Proof: (Structural Induction)

Base Case:

So base case P() holds.

Induction step: Let

Assume holds,

Then

(by induction hypothesis)

(by Hint)

Hence holds.

Then by Structural Induction we get the conclusion. ￭

b)

Define P():

Claim:

Proof: (Structural Induction)

Base Case:

So base case P() holds.

Induction Step: Let

Assume holds,

Want to show that holds, and we have three cases to discuss.

Case 1:

Then

Case 1 holds.

Case 2:

(the added 2 represents left-most ‘((’ OR right-most ‘))’ )

(it depends on which of is not )

(by induction hypothesis)

(the reduced 1 represents the left-most ‘(’ )

Case 2 holds.

Case 3:

Then

(the added 2 represents left-most ‘((’ and right-most ‘))’ )

(by induction hypothesis)

(the reduced 1 represents the left-most ‘(’ )

Case 3 holds.

In all cases holds, then by Structural Induction, we conclude:

￭